

# Quantum gauge symmetry from finite field dependent BRST transformations

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## Abstract

Using the technique of finite field dependent BRST transformations we show that the classical massive Yang-Mills theory and the pure Yang-Mills theory whose gauge symmetry is broken by a gauge fixing term are identical from the view point of quantum gauge symmetry. The explicit infinitesimal transformations which leave the massive Yang-Mills theory BRST invariant are given.

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In a recent paper [1] it was shown that a classical massive gauge theory does not have an essential difference, at the quantum level, from a gauge invariant theory whose gauge symmetry is broken by a gauge fixing term. Specifically, the classical lagrangians,

$$\mathcal{L} = \mathcal{L}_{YM} - \frac{m^2}{2} A_\mu^a A_\mu^a \quad (1)$$

and

$$\mathcal{L} = \mathcal{L}_{YM} - \frac{1}{2} (\partial_\mu A_\mu^a)^2 \quad (2)$$

where  $\mathcal{L}_{YM}$  is the Yang-Mills lagrangian,

$$\mathcal{L}_{YM} = -\frac{1}{4} (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c)^2 \quad (3)$$

could be given an identical physical meaning, both representing an effective gauge fixed lagrangian associated with the quantum theory defined by

$$\int \mathcal{D}A_\mu^a \mathcal{D}B^a \mathcal{D}\bar{c}^a \mathcal{D}c^a \exp \left\{ -S_{YM} + \int d^4x \left[ -iB^a (\partial^\mu A_\mu^a) + \bar{c}^a (-\partial_\mu (D^\mu c)^a) \right] \right\} \quad (4)$$

that is invariant under the BRST transformations,

$$\begin{aligned} \delta A_\mu^a &= -i(D_\mu c)^a \delta\lambda \\ \delta c^a &= -i\frac{g}{2} f^{abc} c^b c^c \delta\lambda \\ \delta \bar{c}^a &= B^a \delta\lambda \\ \delta B^a &= 0 \end{aligned} \quad (5)$$

where  $\delta\lambda$  is an infinitesimal Grassmann parameter. This is to be contrasted with the conventional interpretation of regarding (1) as a massive vector theory and (2) as an effective Yang-Mills theory with a covariant gauge fixing term.

In this paper we shall show the equivalence of the quantum theories defined by (1) and (2) by following the method of finite field dependent BRST (FFBRST) transformations developed by one of us [2]. In particular this method, which will be briefly reviewed below, connects quantum gauge theories in different gauges. Here we start from the conventional

gauge fixed Yang-Mills lagrangian defined by (2). The explicit FFBRST transformations are then stated which maps this theory to one whose lagrangian is defined by (1), thereby showing the connection between them. We also get the form of the transformations that preserve the BRST invariance of the quantum theory defined by (1). Finally we suggest a possible connection between our approach and that adopted in [1], which was based on a modified quantization scheme [3,4], where the variation of the gauge field in the path integral is taken over the entire gauge orbit.

Let us now briefly review the FFBRST approach [2,5–7]. FFBRST transformations are obtained by an integration of infinitesimal (field dependent ) BRST transformations [2]. In this method all the fields are function of some parameter,  $\kappa : 0 \leq \kappa \leq 1$ . For a generic field  $\phi(x, \kappa)$ ,  $\phi(x, \kappa = 0) = \phi(x)$  and  $\phi(x, \kappa = 1) = \phi'(x)$ . Then the infinitesimal field dependent BRST transformations are defined as,

$$\frac{d}{d\kappa}\phi(x, \kappa) = \delta_{BRST}\phi(x, \kappa)\Theta'[\phi(x, \kappa)] \quad (6)$$

where  $\Theta'd\kappa$  is an infinitesimal field dependent parameter. It has been shown by integrating these equations from  $\kappa = 0$  to  $\kappa = 1$  that  $\phi'(x)$  are related to  $\phi(x)$  by FFBRST,

$$\phi'(x) = \phi(x) + \delta_{BRST}\phi(x)\Theta[\phi(x)] \quad (7)$$

where  $\Theta[\phi(x)]$  is obtained from  $\Theta'[\phi(x)]$  through the relation,

$$\Theta[\phi(x)] = \Theta'[\phi(x)] \frac{\exp f[\phi(x)] - 1}{f[\phi(x)]} \quad (8)$$

and  $f$  is given by  $f = \sum_i \frac{\delta\Theta'(x)}{\delta\phi_i(x)}\delta_{BRST}\phi_i(x)$

The choice of the parameter  $\Theta'$  is crucial in connecting different effective gauge theories by means of the FFBRST. In particular the FFBRST of Eq. (7) with  $\Theta'[\phi(x, \kappa)] = i \int \bar{c}^a(y) [F^a[A(\kappa)] - F'^a[A(\kappa)]]$  relates the Yang-Mills theory with an arbitrary gauge fixing  $F[A]$  to the Yang-Mills theory with another arbitrary gauge fixing  $F'[A]$  [5].

The meaning of these field transformations is as follows. We consider the vacuum expectation value of a gauge invariant functional  $G[\phi]$  in some arbitrary gauge  $F[A]$ ,

$$<< G[\phi] >> \equiv \int \mathcal{D}\phi G[\phi] \exp(iS_{eff}^F[\phi]) \quad (9)$$

where,

$$S_{eff}^F = S_0 - \frac{1}{2} \int d^4x F^2[A] - \int d^4x \bar{c}^a W^{ab} c^b \quad (10)$$

with

$$W^{ab} = \frac{\delta F^a}{\delta A_\mu^c} D_\mu^{cb}[A] \quad (11)$$

Here  $S_0$  is the pure Yang-Mills action obtained from (3) and the covariant derivative,  $D_\mu^{ab}[A] \equiv \delta^{ab}\partial_\mu + gf^{abc}A_\mu^c$ . For simplicity we have set the gauge parameter  $\lambda = 1$  in the gauge fixing term  $\frac{1}{2\lambda} \int d^4x F^2[A]$ .

Now we perform the FFBRST transformations  $\phi \rightarrow \phi'$  given by (7). We have then

$$<< G[\phi] >> = << G[\phi'] >> = \int \mathcal{D}\phi' J[\phi'] G[\phi'] \exp(iS_{eff}^F[\phi']) \quad (12)$$

on account of BRST invariance of  $S_{eff}^F$  and gauge invariance of  $G[\phi]$ . Here  $J[\phi']$  is the Jacobian associated with FFBRST and defined as,

$$\mathcal{D}\phi = \mathcal{D}\phi' J[\phi'] \quad (13)$$

As shown in [2] for the special case  $G[\phi] = 1$  the Jacobian  $J[\phi']$  in Eq (12) can always be replaced by  $\exp(iS_1[\phi'])$  with,

$$S_{eff}^F[\phi'] + S_1[\phi'] = S_{eff}^{F'}[\phi'] \quad (14)$$

where

$$S_{eff}^{F'} = S_0 - \frac{1}{2} \int d^4x F'^2[A] - \int d^4x \bar{c}^a W'^{ab} c^b \quad (15)$$

with

$$W'^{ab} = \frac{\delta F'^a}{\delta A_\mu^c} D_\mu^{cb}[A] \quad (16)$$

The extra piece in the action which arises from the Jacobian of such FFBRST is given by,

$$S_1[\phi] = \int d^4x \left[ -\frac{1}{2}F'^2[A] + \frac{1}{2}F^2[A] + \bar{c}[W - W']c \right] \quad (17)$$

Thus the FFBRST in Eq. (7) takes the theory with gauge  $F$  to the corresponding theory with gauge  $F'$ .

We are now ready to apply this machinery to the present problem. We start with the generating functional for the Yang-Mills theory in the Lorentz gauge,

$$Z = \int \mathcal{D}A_\mu \mathcal{D}c \mathcal{D}\bar{c} \exp(iS_{eff}^L) \quad (18)$$

where

$$S_{eff}^L = S_0 - \frac{1}{2} \int d^4x (\partial^\mu A_\mu)^2 - \int d^4x \bar{c} W c \quad (19)$$

with  $W = \partial^\mu D_\mu$  is the Faddeev-Popov determinant. We now apply FFBRST [Eq. (7)] with

$$\Theta' = i \int d^4y \bar{c}^a(y) \left[ \partial^\mu A_\mu^a - m \frac{\omega^\mu}{|\omega|} A_\mu^a \right] (y) \quad (20)$$

where  $\omega^\mu$  is an arbitrary 4-vector, to the expression for the generating functional to obtain,

$$Z = \int \mathcal{D}A'_\mu \mathcal{D}c' \mathcal{D}\bar{c}' \exp i(S_{eff}^L + S_1) \quad (21)$$

The additional piece in the action comes from the non-trivial Jacobian of the FFBRST and can be written using Eq (17)

$$S_1 = \int d^4x \left[ -\frac{1}{2|\omega|^2} m^2 (\omega^\mu A_\mu)^2 + \frac{1}{2} (\partial^\mu A_\mu)^2 - \bar{c}(W' - W)c \right] \quad (22)$$

with  $W' = m \frac{\omega^\mu}{|\omega|} D_\mu$ . Hence we obtain the generating functional for a new effective action given by,

$$S_{eff} = S_0 - \int d^4x \left[ \frac{1}{2} A_\mu M^{\mu\nu} A_\nu + \bar{c} m \frac{\omega^\mu}{|\omega|} D_\mu c \right] \quad (23)$$

where  $M^{\mu\nu}$  is a generalized mass matrix,

$$M^{\mu\nu} = m^2 \frac{\omega^\mu \omega^\nu}{|\omega|^2} \quad (24)$$

This effective action (23) corresponds to the Yang-Mills lagrangian with a generalized mass term. It shows the connection between the Lorentz gauge and a generalized ‘mass’ gauge  $\frac{1}{2}A_\mu M^{\mu\nu} A_\nu$  in the context of Yang-Mills theory. To exactly reproduce the familiar mass term, we restrict the arbitrary vector  $\omega^\mu$  to be of infinitesimal form satisfying the symmetric multiplication rule,

$$\frac{\omega^\mu \omega_\nu}{|\omega|^2} = \frac{g_\nu^\mu}{4} \quad (25)$$

In that case the gauge fixing term is  $\frac{1}{8}m^2 A_\mu A^\mu$  which coincides with the standard mass term, after a proper normalization of  $m$ .

The infinitesimal BRST transformations which leave the Yang-Mills theory with a mass term (23) invariant are given by

$$\begin{aligned} \delta A_\mu^a &= D_\mu^{ab} c^b \delta\lambda \\ \delta c^a &= -\frac{g}{2} f^{abc} c^b c^c \delta\lambda \\ \delta \bar{c}^a &= m \frac{\omega^\mu}{|\omega|} A_\mu^a \delta\lambda \end{aligned} \quad (26)$$

We have shown how, by means of finite field dependent BRST transformations, it was possible to interpolate between the Yang-Mills theory in the covariant gauge to the Yang-Mills theory in a mass like gauge. Since FFBRST also connects the Yang-Mills theory in the axial and covariant gauges [5,6] it is clear that the Yang-Mills theory with a mass like gauge fixing term can also be obtained from other starting points. In this paper we took the covariant gauge as the starting point for reasons of convenience and also comparing our analysis with [1]. It may be pointed out that the latter approach is based on the variation of the gauge variable along the entire gauge orbit, without taking any specific limit of the gauge fixing parameter. Consequently there seems to be a connection between this approach and the FFBRST method, which is not altogether surprising. Carrying out the integration over the complete gauge orbit would be simulated by finite BRST transformations instead

of the conventional infinitesimal one. We feel it might be useful to pursue this connection in a later work.

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